Indian Statistical Institute, Bangalore
B. Math II, First Semester, 2022-23
Back Paper Examination, Introduction to Statistical Inference
21.11.22 **Maximum Score 85** **Duration: 3 Hours**

1. (10) Let $X_1, \cdots, X_n$ be iid observations from population with mean μ and variance σ^2.
   (a) Show that the estimator $\sum_{i=1}^n a_i X_i$ is unbiased for μ iff $\sum_{i=1}^n a_i = 1$
   (b) Among all estimators of the above form find the one with minimum variance.

2. (20) Let ϕ_1 and ϕ_2 be bivariate normal densities with means zero, variances 1 and correlation coefficients 0 and 1/2 respectively. Suppose (X, Y) has the bivariate density $\frac{1}{2}(\phi_1 + \phi_2)$. Find the pdfs of X, Y and $X + Y$. Conclude that (X,Y) is not bivariate normal but the marginals are univariate normal.

3. (5+3+7+5) Let $X_i, \cdots, X_n$ be iid Poisson(λ) random variables.
   (a) Find the method of moments estimator of λ based on the first moment.
   (b) Show that the estimator is consistent.
   (c) Show that $\sqrt{n}(\sqrt{\hat{\lambda}} - \sqrt{\lambda})$ converges in distribution to $\mathcal{N}(0, 1/4)$.
   (d) Find an asymptotic $(1 - \alpha)$ level confidence interval for λ using (c).

4. (5+5+5+5) Let $X_1, X_2, \cdots, X_n$ be a random sample from $\mathcal{N}(5, \sigma^2)$ where σ^2 is unknown.
   (a) Consider testing $H_0 : \sigma^2 = 1$ versus $H_1 : \sigma^2 = 2$. Find the most powerful test at level 5%.
   (b) Show that the family of densities possesses Monotone Likelihood Ratio property.
   (c) Derive Uniformly Most Powerful test of level 5% for testing $H_0 : \sigma^2 \leq 1$ versus $H_1 : \sigma^2 > 1$.
   (d) Is the test in part (c) unbiased? Justify your answer.

5. (15) Let $X_1, \cdots X_n$ be iid Unif$(\theta, 2\theta)$. Find the generalized likelihood ratio test for testing $H_0 : \theta = \theta_0$ vs $H_1 : \theta \neq \theta_0$ at level α. Find the critical region explicitly in terms of α, n and θ_0.